Funciones de 1 variable

Ejercicio Calcule un valor aproximado de $\sqrt[5]{1,03}$ cometiendo un error menor de 10^{-5}.

Consideremos la función $f(x) = x^{1/5}$. Lo que tenemos que hacer es buscar una aproximación de f en el punto $1,03$.

Así que vamos a hacer el desarrollo de Taylor de f centrado en $a = 1$. Para conocer el orden tenemos que imponer la condición del error $< 10^{-5}$.

$$f(x) = x^{1/5}$$

$$f'(x) = \frac{1}{5} x^{-4/5}$$

$$f''(x) = -\frac{1}{5} \frac{4}{5} \cdot x^{-9/5}$$

$$f'''(x) = \frac{1}{5} \cdot \frac{4}{5} \cdot \frac{9}{5} x^{-14/5}$$

así que es fácil ver que:

$$\boxed{f^{(n)}(x) = (-1)^{n+1} \frac{\prod_{j=1}^{n-1} (5j-1)}{5^n} x^{-\frac{5n-1}{5}}} \quad \text{para } n > 1$$

Si aproximamos el valor de $f(1,03)$ por el de $P_n(1,03)$ el error que se comete es:

$$E = \left| R_{n+1}(1,03) \right| = \left| \frac{f^{(n+1)}(\xi)}{(n+1)!} \cdot (1,03-1)^{n+1} \right| =$$

$$\left| f^{(n+1)}(\xi) \right| \frac{0,03^{n+1}}{(n+1)!} = \frac{\prod_{j=1}^{n} (5j-1)}{5^{n+1}} \xi^{-\frac{5n+4}{5}} \cdot \frac{3^{n+1} \cdot 10^{-2n-2}}{(n+1)!}$$

para algún $\xi \in (1, 1.003)$.

Observa que $g(\xi) = \xi^{-\frac{5n+4}{5}}$ es decreciente en $(1, 1,003)$ porque $g' < 0$.

Luego:

$$\underbrace{E < \frac{\prod_{j=1}^{n} (5j-1)}{5^{n+1}} 1^{-\frac{5n+4}{5}} \cdot \frac{3^{n+1} 10^{-2n-2}}{(n+1)!} \overset{?}{<} 10^{-5}}$$

$$\Updownarrow$$

$$\underbrace{\prod_{j=1}^{n} (5j-1) \cdot 3^{n+1} 10^{-2n-2}}_{S_n} \overset{?}{<} \underbrace{10^{-5} (n+1)!\, 5^{n+1}}_{T_n}$$

n	S_n	T_n	$S_n \overset{?}{<} T_n$
1	$36 \cdot 10^{-4}$	$50 \quad 10^{-5} = 5 \cdot 10^{-4}$	NO
2	$972 \cdot 10^{-6}$	$750 \cdot 10^{-5} = 7500\ 10^{-6}$	SÍ

Por lo tanto nesesitamos el polinomio de grado 2 centrado en $a=1$:

$$P_2(x) = 1 + \frac{1}{5}(x-1)$$

$$\sqrt[5]{1{,}03} \simeq 1 + \frac{1}{5}\, 0{,}03 \qquad y \quad |E| < 10^{-5}$$

7.2

a) $f(x) = e^x \qquad a = 0$

$$f(x) = f'(x) = f''(x) = \dots = f^{(k)}(x) = e^x$$

$$f(0) = f'(0) = f''(0) = \dots = f^{(k)}(0) = 1$$

$$P_k(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \dots + \frac{f^{(k)}(0)}{k!}x^k =$$

$$= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^k}{k!}$$

b) $f(x) = \operatorname{sen} x \qquad a = 0$

$$\begin{aligned} f(x) &= \operatorname{sen} x = f^{(iv)}(x) = f^{(viii)}(x) \\ f'(x) &= \cos x = f^{(v)}(x) \quad \vdots \\ f''(x) &= -\operatorname{sen} x = f^{(vi)}(x) \quad \vdots \\ f'''(x) &= -\cos x = f^{(vii)}(x) \quad \vdots \end{aligned}$$

$$f(0) = 0 = f^{(4k)}(0) \qquad \forall k \in \mathbb{N} \cup \{0\}$$

$$f'(0) = 1 = f^{(4k+1)}(0)$$

$$f''(0) = 0 = f^{(4k+2)}(0)$$

$$f'''(0) = -1 = f^{(4k+3)}(0)$$

$$P_{2k+1}(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots + (-1)^k \frac{x^{2k+1}}{(2k+1)!}$$

7.2 c) $f(x) = \cos x$

Como $\operatorname{sen}'(x) = \cos x$, la derivada del desarrollo del seno es el desarrollo del coseno:

$$P_{2k}(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots + (-1)^k \frac{x^{2k}}{(2k)!}$$

d) $f(x) = \frac{1}{1+x}$

$$f'(x) = \frac{-1}{(1+x)^2}$$

$$f''(x) = \frac{2(1+x)}{(1+x)^4} = \frac{2}{(1+x)^3} \quad ; \quad f'''(x) = \frac{-3\cdot 2\cdot (1+x)^2}{(1+x)^6} = \frac{-3!}{(1+x)^4}$$

$$f^{(iv)}(x) = \frac{+4\cdot 3!\,(1+x)^3}{(1+x)^8} = \frac{4!}{(1+x)^5}$$

Así que se puede demostrar por inducción que:

$$f^{(n)}(x) = \frac{(-1)^n \cdot n!}{(1+x)^{n+1}}$$

Y ahora:

$$f(0) = 1; \; f'(0) = -1 \; ; \; f''(0) = 2 \; ; \; f^{(iii)}(0) = -3! \; \dots \; f^{(n)}(0) = (-1)^n n!$$

Finalmente:

$$P_n(x) = 1 - x + x^2 - x^3 + x^4 - x^5 + \dots + (-1)^k x^k$$

7.2. e) $f(x) = \log(1+x)$

Como $\log(1+x) = \int \frac{1}{1+x}\,dx$, basta con hacer la primitiva del desarrollo de $\frac{1}{1+x}$ para obtener el de $\log(1+x)$:

$$P_k(x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} + \cdots + (-1)^{k+1}\frac{x^k}{k}$$

f) $f(x) = \cos(x^2)$

Componiendo el desarrollo de $\cos(x)$ con x^2 obtenemos el desarrollo de $\cos(x^2)$.

Como

$$P_{2k}^{\cos}(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots + (-1)^k \frac{x^{2k}}{(2k)!}$$

$$P_{4k}(x) = 1 - \frac{x^4}{2!} + \frac{x^8}{2!} - \frac{x^{12}}{6!} + \cdots + (-1)^k \frac{x^{4k}}{(2k)!}$$

7.3.a) $\sqrt[4]{e} = ?$ usando un polinomio de Taylor de orden 3.

$$f(x) = e^x$$

$$P_3(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$$

$$e^{1/4} = f(1/4) \simeq 1 + \frac{1}{4} + \frac{1}{32} + \frac{1}{384}$$

$$E = |R_4(1/4)| = \left| \frac{f^{(iv)}(\xi)}{4!} \left(\frac{1}{4}\right)^4 \right| = \frac{e^{\xi}}{4^4 \cdot 24} < \frac{e^{1/4}}{4^4 \cdot 24} < \frac{e^1}{4^4 \cdot 24}$$

$\xi \in (0, \frac{1}{4})$

$$< \frac{3}{4^4 \cdot 24} = \frac{1}{2^3 \cdot 4^4} = \frac{1}{2^{11}}$$

b) $\cos(0{,}5)$ $\qquad f(x) = \cos x$

$$P_3(x) = 1 - \frac{x^2}{2}$$

$$\cos(0{,}5) = f(0{,}5) \simeq 1 - \frac{0.5^2}{2}$$

seno o coseno con signo $\pm$

$$E = |R_4(0{,}5)| = \left| \frac{f^{(iv)}(\xi)}{4!} 0{,}5^4 \right| = \frac{|f^{(iv)}(\xi)|}{4!} 0{,}5^4 \leq \frac{0{,}5^4}{4!}$$

$\xi \in (0, 0.5)$

c) $\log(1.5)$ $\qquad f(x) = \log(1+x)$

$$P_3(x) = x - \frac{x^2}{2} + \frac{x^3}{3}$$

$$\log(1{,}5) = f(0{,}5) \simeq 0{,}5 - \frac{0{,}5^2}{2} + \frac{0{,}5^3}{3}$$

$$f'(x) = \frac{1}{1+x} \qquad f''(x) = \frac{-1}{(1+x)^2} \qquad f'''(x) = \frac{2}{(1+x)^3} \qquad f^{(iv)}(x) = \frac{-6}{(1+x)^4}$$

$$E = |R_4(0{,}5)| = \left| \frac{f^{(iv)}(\xi)}{4!} \, 0{,}5^4 \right| = \underbrace{\frac{1}{(1+\xi)^4}}_{g(\xi)} \frac{0{,}5^4 \cdot 6}{4!}$$

$\xi \in (0, 0{.}5)$

Como $$g'(\xi) = \frac{-4(1+\xi)^3}{(1+\xi)^8} = \frac{-4}{(1+\xi)^5} < 0$$ g es decreciente y

$$E < \frac{0{,}5^4 \cdot 6}{4!}$$

7.5

$$\lim_{x\to 0} \frac{\operatorname{sen}^3(x^2)}{1-\cos(x^3)}$$

Hacemos el desarrollo de orden 6:

$$\operatorname{sen} x = x - \frac{x^3}{3!} + \frac{x^6}{6!} + o(x^6)$$

$$\operatorname{sen}(x^2) = x^2 - \frac{x^6}{3!} + o(x^6)$$

$$\operatorname{sen}^3(x^2) = x^6 + o(x^6)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + o(x^6)$$

$$\cos x^3 = 1 - \frac{x^6}{2!} + o(x^6)$$

$$1-\cos(x^3) = \frac{x^6}{2!} + o(x^6)$$

$$\lim_{x\to 0} \frac{\operatorname{sen}^3(x^2)}{1-\cos(x^3)} = \lim_{x\to 0} \frac{x^6 + o(x^6)}{\frac{x^6}{2!} + o(x^6)} =$$

$$= \lim_{x\to 0} \frac{1 + \frac{o(x^6)}{x^6}}{\frac{1}{2} + \frac{o(x^6)}{x^6}} = 2$$

7.6 $$\lim_{x\to 0} \frac{[1-\cos^2(x^2)]\,\operatorname{sen}^3(x)}{x\log(1+x^6)}$$

Hacemos desarrollos de orden 7:

$$\operatorname{sen} x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + o(x^7)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + o(x^7)$$

$$\cos(x^2) = 1 - \frac{x^4}{2!} + o(x^7)$$

$$\cos^2(x^2) = 1 - \frac{2x^4}{2!} + o(x^7) = 1 - x^4 + o(x^7)$$

$$1-\cos^2(x^2) = x^4 + o(x^7)$$

el término de orden 3 es 0

$$\operatorname{sen}^2(x) = x^2 + Ax^4 + Bx^5 + Cx^6 + Dx^7 + o(x^7)$$

$$\operatorname{sen}^3(x) = x^3 + ax^4 + bx^5 + cx^6 + dx^7 + o(x^7)$$

$$[1-\cos^2(x^2)]\operatorname{sen}^3(x) = x^7 + o(x^7)$$

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \frac{x^6}{6} + \frac{x^7}{7} + o(x^7)$$

$$\log(1+x^6) = x^6 + o(x^7)$$

$$x\log(1+x^6) = x^7 + o(x^7)$$

$$\lim_{x\to 0} \frac{[1-\cos^2(x^2)]\operatorname{sen}^3(x)}{x\log(1+x^6)} = \lim_{x\to 0}\frac{x^7+o(x^7)}{x^7+o(x^7)} = \lim_{x\to 0}\frac{1+\frac{o(x^7)}{x^7}}{1+\frac{o(x^7)}{x^7}}$$

$$= 1$$

7.7

$$\lim_{x\to 0}\left(\frac{1}{\operatorname{sen}^2 x}-\frac{1}{x^2}\right)=\lim_{x\to 0}\frac{x^2-\operatorname{sen}^2 x}{x^2\operatorname{sen}^2 x}=L$$

$$\operatorname{sen} x = x-\frac{x^3}{3!}+o(x^4)$$

$$\operatorname{sen}^2 x = x^2-\frac{1}{3}x^4+o(x^4)$$

$$x^2-\operatorname{sen}^2 x=\frac{1}{3}x^4+o(x^4)$$

$$x^2\operatorname{sen}^2 x = x^4+o(x^4)$$

$$L=\lim_{x\to 0}\frac{\frac{1}{3}x^4+o(x^4)}{x^4+o(x^4)}=\lim_{x\to 0}\frac{\frac{1}{3}+\frac{o(x^4)}{x^4}}{1+\frac{o(x^4)}{x^4}}=\frac{1}{3}$$

7.9

$$\lim_{x\to 0} \frac{\log^3(1-3x^2)}{x^4 - x^4\cos^2(2x)}$$

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \frac{x^6}{6} + o(x^6)$$

$$\log(1-3x^2) = -3x^2 - \frac{9x^4}{2} - \frac{27x^6}{3} + o(x^6)$$

$$\log^2(1-3x^2) = 9x^4 + \frac{27}{2}x^6 + o(x^6)$$

$$\log^3(1-3x^2) = -27x^6 + o(x^6)$$

$$\cos(x) = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} + o(x^6)$$

$$\cos(2x) = 1 - \frac{4x^2}{2} + \frac{16x^4}{4!} - \frac{64x^6}{6!} + o(x^6)$$

$$\cos^2(2x) = 1 - 4x^2 + Ax^3 + Bx^4 + Cx^5 + Dx^6 + o(x^6)$$

$$x^4\cos^2(2x) = x^4 - 4x^6 + o(x^6)$$

$$x^4 - x^4\cos^2(2x) = 4x^6 + o(x^6)$$

$$\lim_{x\to 0} \frac{\log^3(1-3x^2)}{x^4 - x^4\cos^2(2x)} = \lim_{x\to 0} \frac{-27x^6 + o(x^6)}{4x^6 + o(x^6)} =$$

$$= \lim_{x\to 0} \frac{-27 + \frac{o(x^6)}{x^6}}{4 + \frac{o(x^6)}{x^6}} = \frac{-27}{4}$$

7.10 $f(x) = \int_0^x te^{-t}\,dt$

$$\int_0^{0.1} te^{-t}\,dt \qquad \boxed{a=0}$$

$$f'(x) = xe^{-x} \quad ; \quad f''(x) = e^{-x} - xe^{-x}$$

$$f'''(x) = -e^{-x} - e^{-x} + xe^{-x} = -2e^{-x} + xe^{-x}$$

$$P_2(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 = \frac{1}{2}x^2$$

$$\int_0^{0.1} te^{-t} \approx P_2(0.1) = \frac{10^{-2}}{2}$$

$$E = |R_3(0.1)| = \left| \frac{f'''(\xi)}{3!} 0.1^3 \right| = \frac{|-2e^{-\xi} + \xi e^{-\xi}|}{3!} 10^{-3} \leq$$

$\xi \in (0, 0.1)$

$$\leq \frac{|-2e^{-\xi}| + |\xi e^{-\xi}|}{3!} 10^{-3} = \frac{2e^{-\xi} + \xi e^{-\xi}}{3!} 10^{-3} \leq$$

$$\leq \frac{2e^{0} + 0.1 \cdot e^{0}}{3!} 10^{-3} < \frac{2.1}{3!} 10^{-3}$$

7.11 $\lim_{x\to 0} \dfrac{1-\cos(3x)}{\operatorname{sen}(x^2)}$

Aquí hacemos un desarrollo de orden x^2:

$$\cos(x) = 1 - \frac{x^2}{2} + o(x^2)$$

$$\cos(3x) = 1 - \frac{9x^2}{2} + o(x^2) \;; \qquad 1-\cos(3x) = \frac{9x^2}{2} + o(x^2)$$

$$\operatorname{sen} x = x + o(x^2)$$

$$\operatorname{sen} x^2 = x^2 + o(x^2)$$

$$\lim_{x\to 0} \frac{1-\cos(3x)}{\operatorname{sen}(x^2)} = \lim_{x\to 0} \frac{\frac{9x^2}{2} + o(x^2)}{x^2 + o(x^2)} = \lim_{x\to 0} \frac{9/2 + \frac{o(x^2)}{x^2}}{1 + \frac{o(x^2)}{x^2}} = \frac{9}{2}$$

7.12 $\lim\limits_{x\to 0} \dfrac{\operatorname{sen} x - \dfrac{x+ax^3}{1+bx^2}}{\log 1+x} = \lim\limits_{x\to 0} \dfrac{(1+bx^2)\operatorname{sen} x - x - ax^3}{(1+bx^2)\log(1+x)}$

$\operatorname{sen}(x) = x + o(x)$

$(1+bx^2)\operatorname{sen} x - x - ax^3 = o(x)$

$\log(1+x) = x + o(x)$

$(1+bx^2)\log(1+x) = x + o(x)$

$$\lim_{x\to 0} \frac{(1+bx^2)\operatorname{sen} x - x - ax^3}{(1+bx^2)\log(1+x)} = \lim_{x\to 0} \frac{o(x)}{x+o(x)} = \lim_{x\to 0} \frac{o(x)/x}{1+\frac{o(x)}{x}}$$

$= 0$

7.11

$$\lim_{x\to 0} \frac{1-\cos(3x)}{\operatorname{sen} x^2} = \lim_{x\to 0} \frac{\frac{9}{2}x^2 + o(x^2)}{x^2 + o(x^2)} = \lim_{x\to 0} \frac{\frac{9}{2} + \frac{o(x^2)}{x^2}}{1 + \frac{o(x^2)}{x^2}} = \frac{9}{2}$$

$$\cos x = 1 - \frac{x^2}{2} + o(x^2)$$

$$\cos(3x) = 1 - \frac{9x^2}{2} + o(x^2) \qquad 1-\cos(3x) = \frac{9x^2}{2} + o(x^2)$$

$$\operatorname{sen} x = x + o(x^2)$$

$$\operatorname{sen}(x^2) = x^2 + o(x^2)$$

7.15 d $\lim_{x \to 0} \frac{\log(1+x^3)}{1-\cos^3 x} = L$

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + o(x^3)$$

$$\log(1+x^3) = x^3 + o(x^3)$$

$$\cos x = 1 - \frac{x^2}{2} + o(x^3)$$

$$\cos^3 x = 1 - \frac{3}{2}x^2 + o(x^3)$$

$$1-\cos^3 x = \frac{3}{2}x^2 + o(x^3)$$

$$L = \lim_{x \to 0} \frac{x^3 + o(x^3)}{\frac{3}{2}x^2 + o(x^3)} = \lim_{x \to 0} \frac{x + \frac{o(x^3)}{x^2}}{\frac{3}{2} + \frac{o(x^3)}{x^2}} = 0$$

Ejercicio Calcula una aproximación de sen(0.01) cometiendo un error menor que 10^{-7}

$f(x) = \text{sen}(x)$ $\qquad$ $p_n(x)$

$a = 0$

$$f(x) \simeq p_n(x) \qquad x = 0.01$$

$n \to ?$

$$E = |f(x) - p_n(x)| = |R_{n+1}(x)| =$$

$$\left| \frac{f^{(n+1)}(\xi)}{(n+1)!}(x-0)^{n+1} \right| = \frac{|f^{(n+1)}(\xi)|}{(n+1)!}(0.01)^{n+1}$$

$$\leq \frac{\left| \begin{matrix} \pm \operatorname{sen} \xi \\ \circ \\ \pm \cos \xi \end{matrix} \right| 10^{-2n-2}}{(n+1)!} \leq \boxed{\frac{1 \; 10^{-2n-2}}{(n+1)!}}$$

$Q(n)$

$$\overset{?}{<} 10^{-7}$$

Investigamos el valor de n que necesito:

n	$Q(n)$	$Q(n) < 10^{-7}$
1	$\frac{10^{-4}}{2}$	NO
2	$\frac{10^{-6}}{6}$	NO
3	$\frac{10^{-8}}{4!}$	SÍ

Hacemos el desarrollo de Taylor de orden 3 del $sen(x)$ centrado en $a=0$

$$f(x) = sen(x) = f^{(iv)}(x)$$
$$f'(x) = \cos x$$
$$f''(x) = -\operatorname{sen} x$$
$$f'''(x) = -\cos x$$

$$f(0) = 0 = f^{(4)}(x)$$
$$f'(0) = 1 = f^{(5)}(x)$$
$$f''(0) = 0$$
$$f'''(0) = -1$$

$$P_3(x) = 0 + x + \frac{0}{2!}x^2 - \frac{1}{3}x^3$$

$$\text{sen}(0.01) = f(0.01) \simeq p_3(0.01) =$$

$$= 0.01 - \frac{0.01^3}{3!}$$

Ejercicio: Calcular una aproximación de $e^{1/5}$ cometiendo un error menor que 10^{-5}.

Usamos la función $f(x) = e^x$ para aproximar $f(1/5) = e^{1/5} = \sqrt[5]{e}$. Hacemos el polinomio centrado en $a=0$.

Así que $f(x) = P_n(x) + R_{n+1}(x)$ y

$$E = |f(x) - P_n(x)| \quad \text{con } x = 1/5$$

$$E = |R_{n+1}(x)| = \left| \frac{f^{(n+1)}(\xi)}{(n+1)!} x^{n+1} \right| =$$

$$= \frac{e^{\xi}}{(n+1)!} x^{n+1} \qquad \xi \in (\underbrace{0}_{a}, \underbrace{1/5}_{x})$$

Así que

$$E = \frac{e^{\xi}}{(n+1)!}\,\frac{1}{5^{n+1}} \le \frac{e^{1/5}}{(n+1)!}\,\frac{1}{5^{n+1}} \le$$

$$\le \frac{e^{1}}{(n+1)!}\,\frac{1}{5^{n+1}} \le \underbrace{\frac{3}{(n+1)!\,5^{n+1}}}_{Q(n)} \overset{?}{<} 10^{-5}$$

Investigamos el valor de n para el que se cumple la desigualdad:

n	$Q(n)$	$Q(n) \overset{?}{<} 10^{-5}$
1	$\frac{3}{2 \cdot 25} = \frac{3}{50}$	NO
2	$\frac{3}{6 \cdot 5^3} = 0.04$	NO
3	$2 \cdot 10^{-4}$	NO
4	$8 \cdot 10^{-6} = 0.8 \cdot 10^{-5}$	SÍ

Así que basta con hacer un polinomio de Taylor de orden 4

$$P_4(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^{(4)}(0)}{4!}x^4 = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!}$$

Y la aproximación será:

$$e^{1/5} = \sqrt[5]{e} \simeq P_4(1/5) = 1 + \frac{1}{5} + \frac{(1/5)^2}{2} + \frac{(1/5)^3}{3!} + \frac{(1/5)^4}{4!} = \frac{6107}{5000} = 1.2214$$

Examen final GIC junio 2017
Parte del 1er parcial

1) $\beta = \{\overset{v_1}{(1,0,0)}, \overset{v_2}{(1,0,2)}, \overset{v_3}{(0,1,5)}\}$

$\beta' = \{\overset{w_1}{(1,1,1)}, \overset{w_2}{(0,1,5)}, \overset{w_3}{(0,0,1)}\}$

$$M_{\beta\beta'} = \begin{array}{c} \begin{pmatrix} 3 & 0 & -1/2 \\ -2 & 0 & 1/2 \\ 1 & 1 & 0 \end{pmatrix} \\ \begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ w_1 & w_2 & w_3 \end{array} \end{array}$$

$w_1 = (1,1,1) = \alpha v_1 + \beta v_2 + \gamma v_3 = v_3 - 2v_2 + 3v_1$

$w_2 = (0,1,5) = v_3 = (0,0,1)_\beta$

$w_3 = (0,0,1) = \frac{1}{2} v_2 - \frac{1}{2} v_1 = \left(-\frac{1}{2}, \frac{1}{2}, 0\right)_\beta$

También lo podemos hacer de la siguiente forma:

$$M_{\beta\beta'} = M_{\beta\beta_c} M_{\beta_c\beta'} =$$

$$= \left(M_{\beta_c\beta}\right)^{-1} M_{\beta_c\beta'} =$$

$$= \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & 5 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 5 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 0 & -1/2 \\ -2 & 0 & 1/2 \\ 1 & 1 & 0 \end{pmatrix}$$

② $\beta = \{\underset{v_1}{(1,1,1)}, \underset{v_2}{(1,0,1)}, \underset{v_3}{(0,0,1)}\}$

$f: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$

$$M_{\beta\beta}(f) = \begin{array}{c} \begin{array}{ccc} f(v_1) & f(v_2) & f(v_3) \end{array} \\ \begin{pmatrix} 0 & 0 & -1 \\ 3 & 1 & 8 \\ 0 & 0 & 1 \end{pmatrix} \end{array}$$

Ⓐ Fórmula:

$$M_{\beta_c\beta_c}(f) = M_{\beta_c \beta} \, M_{\beta\beta}(f) \, M_{\beta \beta_c} =$$

$$= \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & -1 \\ 3 & 1 & 8 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix}^{-1}$$

$$= \begin{pmatrix} -6 & 2 & 7 \\ 1 & 0 & -1 \\ -7 & 2 & 8 \end{pmatrix}$$